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Approximate Disentanglement

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We construct a quantum machine which, by using an asymmetric cloner, disentangles an entangled state and nearly retains it. The attainable maximum value of the scaling parameter for disentangling is identical to that obtained in previous works. The fidelity of the output residual entangled state with respect to the input entangled state is state-dependent. The result shows that it is possible to deal with disentanglement and broadcasting entanglement in a single unitary evolution.

KEY WORDS: disentanglement; asymmetric cloning.

Quantum entanglement plays an important role in the quantum information field. Manipulation of entanglement such as purification (Bennett *et al.*, 1996; Deutsch *et al.*, 1996) and broadcasting (Bandyopadhyay and Kar, 1999; Buzek *et al.*, 1997) is an intriguing issue in the research on entanglement. Recently disentanglement has attracted a lot of attention (Bandyopadhyay *et al.*, 1999; Feng *et al.*, 2001a,b; Ghosh *et al.*, 2000, 2001; Mor, 1999; Terno, 1999; Zhou and Guo, 2000). Disentanglement is a process in which an initial entangled state of a composite system can be transformed into a separable state without affecting the reduced density matrices of the subsystems. However like other "no-go" theorems (e.g., the no-cloning theorem (Wootters and Zurek, 1982), no-deleting theorem (Pati and Braunstein, 2000), no-broadcasting theorem (Barnum *et al.*, 1996), perfect disentanglement is prohibited by elementary rules of quantum mechanics (Mor, 1999; Terno, 1999). But dropping the constraint that the reduced density matrices of subsystems are perfectly unaffected, approximate disentanglement can be realized by local operations (Ghosh *et al.*, 2000; Zhou and Guo, 2000), e.g., by local cloning

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(Bandyopadhyay *et al.*, 1999) and by teleportation via separable channels (Ghosh *et al.*, 2001). In other words, for a two-qubit entangled state, the transformation

$$\rho^{\mathrm{ent}} \rightarrow \rho^{\mathrm{disent}} \tag{1}$$

can be achieved together with

$$\mathrm{Tr}_j(\rho^{\mathrm{disent}}) = s_i \mathrm{Tr}_j(\rho^{\mathrm{ent}}) + \left(\frac{1-s_i}{2}\right) I, \quad i \neq j; \;\; i, j = 1, 2 \tag{2}$$

for all ρ^{ent}, where s_i ($0 < s_i < 1$ for $i = 1, 2$) is a scaling parameter indendent of ρ^{ent}, standing as a measure of closeness between the ith-reduced-density matrix before and after the transformation. The attainable values of s_1 and s_2 satisfy the inequality $s_1 s_2 \leq \frac{1}{3}$. More explicitly, if only one party undergoes local operation, i.e., $s_1 = 1$ (or $s_2 = 1$), the maximum value of s_2 (or s_1) is $\frac{1}{3}$; if both two parties undergo the same local operation separately, the maximum value of s_2 (or s_1) is $\frac{1}{\sqrt{3}}$. The schemes of realization for both cases are concretely proposed in (Bandyopadhyay *et al.*, 1999; Ghosh *et al.*, 2000).

In this short note, we substitute an asymmetric (isotropic) cloner for the symmetric (isotropic) cloner in the schemes of disentanglement by local cloning (Bandyopadhyay *et al.*, 1999). Then a quantum machine is constructed, which can implement the disentanglement of an entangled state and almost retain it.

Our quantum machine is based on an asymmetric $1 \rightarrow 2$ cloning. The asymmetric cloning in general is described by a pauli channel (Cerf, 1988, 2000). But, for convenience, here the action of the asymmetric cloner is specified by a particular unitary transformation on the state $|\varphi\rangle = \alpha|0\rangle + \beta|1\rangle$ of the input qubit a, where α and β are unknown real parameters, $\alpha^2 + \beta^2 = 1$ and $0 \leq \alpha \cdot \beta \leq 1$. From orthogonality, unitarity and isotropy, this transformation can be presented as

$$|0\rangle_a|\xi\rangle_b|Q\rangle_c \rightarrow \frac{1}{\sqrt{N}}|00\rangle_{ab}|\uparrow\rangle_c + \left(\frac{p}{\sqrt{N}}|01\rangle_{ab} + \frac{q}{\sqrt{N}}|10\rangle_{ab}\right)|\downarrow\rangle_c \tag{3}$$

$$|1\rangle_a|\xi\rangle_b|Q\rangle_c \rightarrow \frac{1}{\sqrt{N}}|11\rangle_{ab}|\downarrow\rangle_c + \left(\frac{p}{\sqrt{N}}|10\rangle_{ab} + \frac{q}{\sqrt{N}}|01\rangle_{ab}\right)|\uparrow\rangle_c, \tag{4}$$

where N is a normalization factor given by $N = 1 + p^2 + q^2, q = 1 - p$, $p > q$, $|Q\rangle_c$ describes the initial state of the cloner, $|\xi\rangle_b$ is an arbitrary initial state of qubit b, and $|\uparrow\rangle_c$ and $|\downarrow\rangle_c$ are two orthonormal vectors in the Hilbert space of the cloner. The asymmetric cloning transformation outputs two copies of the state $|\varphi\rangle$ on the qubits a and b,

$$\rho_a = \frac{2p}{N}|\varphi\rangle\langle\varphi| + \left(1 - \frac{2p}{N}\right)\frac{I}{2}, \tag{5}$$

$$\rho_b = \frac{2q}{N}|\varphi\rangle\langle\varphi| + \left(1 - \frac{2q}{N}\right)\frac{I}{2}, \tag{6}$$

where I is unitary matrix. So for the qubits a and b, the reduction factors by which the cloner shrinks the vector characterizing the input state in the Bloch sphere are $\eta_a = \frac{2p}{N}$ and $\eta_b = \frac{2q}{N}$, respectively. Now that $p \neq q$, the asymmetric cloner outputs two copies with different fidelities for all input states. And

$$\eta_a^2 + \eta_b^2 + \eta_a \eta_b - \eta_a - \eta_b = 0, \tag{7}$$

which satisfies the no-cloning inequality deduced from Eq. (6) in Cerf (2000). Therefore the distribution of information at the outputs of the cloner is controlled via changing the value of p (Buzek *et al.*, 1998).

In the following we consider how the quantum machine achieves the goal of disentangling an entangled state and retaining it approximately. The goal is achieved by applying the above asymmetric $1 \to 2$ cloning to copy both qubits separately.

Suppose qubits a_I and a_{II} share an entangled state

$$|\chi\rangle = \alpha|00\rangle_{a_I a_{II}} + \beta|11\rangle_{a_I a_{II}}, \tag{8}$$

where α and β are defined as before. Both two qubits in the state $|\chi\rangle$ are cloned according to the transformation defined by Eqs. (3) and (4) separately. Then the original state $|\chi\rangle$ is splitted into two branches: two copies $\rho^{\text{out}}_{a_I a_{II}}$ and $\rho^{\text{out}}_{b_I b_{II}}$ of the entangled state $|\chi\rangle$ are produced. What we want to do is to obtain a disentangling state of the state $|\chi\rangle$ and maintain it nearly. We check the inseparability of two copies $\rho^{\text{out}}_{a_I a_{II}}$ and $\rho^{\text{out}}_{b_I b_{II}}$. The output density matrices $\rho^{\text{out}}_{a_I a_{II}}$ and $\rho^{\text{out}}_{b_I b_{II}}$ are given by

$$\begin{aligned}\rho^{\text{out}}_{a_I a_{II}} = {} & \left(\frac{(1+p^2)^2}{N^2}\alpha^2 + \frac{q^4}{N^2}\beta^2\right)|00\rangle\langle 00| + \left(\frac{(1+p^2)^2}{N^2}\beta^2 + \frac{q^4}{N^2}\alpha^2\right)|11\rangle\langle 11| \\ & + \frac{(1+p^2)q^2}{N^2}|01\rangle\langle 01| + \frac{(1+p^2)q^2}{N^2}|10\rangle\langle 10| \\ & + 4\frac{p^2}{N^2}\alpha\beta|00\rangle\langle 11| + 4\frac{p^2}{N^2}\alpha\beta|11\rangle\langle 00| \end{aligned} \tag{9}$$

$$\begin{aligned}\rho^{\text{out}}_{b_I b_{II}} = {} & \left(\frac{(1+q^2)^2}{N^2}\alpha^2 + \frac{p^4}{N^2}\beta^2\right)|00\rangle\langle 00| + \left(\frac{(1+q^2)^2}{N^2}\alpha^2 + \frac{p^4}{N^2}\beta^2\right)|11\rangle\langle 11| \\ & + \frac{(1+q^2)p^2}{N^2}|01\rangle\langle 01| + \frac{(1+q^2)p^2}{N^2}|10\rangle\langle 10| \\ & + 4\frac{q^2}{N^2}\alpha\beta|00\rangle\langle 11| + 4\frac{q^2}{N^2}\alpha\beta|11\rangle\langle 00|. \end{aligned} \tag{10}$$

It follows from Peres–Horodecki theorem (Horodecki, 1996; Peres, 1996) that if $\frac{1-\sqrt{3}+\sqrt{2\sqrt{3}}}{2} \leq p \leq 1$, $\rho^{\text{out}}_{a_I a_{II}}$ is inseparable for

$$\frac{1}{2} - \sqrt{\frac{1}{4} - \left(\frac{(1+p^2)(1-p)^2}{4p^2}\right)^2} \leq \alpha^2 \leq \frac{1}{2} + \sqrt{\frac{1}{4} - \left(\frac{(1+p^2)(1-p)^2}{4p^2}\right)^2}, \tag{11}$$

however $\rho^{\text{out}}_{b_I b_{II}}$ is separable for any values of α^2. So by choosing appropriate value of p of the asymmetric cloner, a residual state $\rho^{\text{out}}_{a_I a_{II}}$ close to the state $\rho_{a_I a_{II}} = |\chi\rangle\langle\chi|$ and a disentangling state $\rho^{\text{out}}_{b_I b_{II}}$ can be obtained in a single evolution.

For the residual state $\rho^{\text{out}}_{a_I a_{II}}$, the fidelity with respect to the original entangled state $|\chi\rangle$ is examined. The fidelity is defined as

$$F = \langle\chi|\rho^{\text{out}}_{a_I a_{II}}|\chi\rangle = \frac{(1+p^2)^2}{N^2} - \frac{8pq^2}{N^2}|\alpha|^2|\beta|^2. \tag{12}$$

Obviously, the fidelity F is dependent on the input entangled state $|\chi\rangle$. For the disentangling state $\rho^{\text{out}}_{b_I b_{II}}$, the factors s of qubits b_I and b_{II} are inspected.

$$\begin{aligned}\rho^{\text{out}}_{b_I} &= \text{Tr}_{b_{II}}\left(\rho^{\text{out}}_{b_I b_{II}}\right) \\ &= \frac{2q}{N}(\alpha^2|0\rangle\langle 0| + \beta^2|1\rangle\langle 1|) + \frac{p^2}{N}(|0\rangle\langle 0| + |1\rangle\langle 1|) \\ &= \frac{2q}{N}\text{Tr}_{a_{II}}(|\chi\rangle\langle\chi|) + \frac{p^2}{N}I, \end{aligned} \tag{13}$$

$$\rho^{\text{out}}_{b_{II}} = \frac{2q}{N}\text{Tr}_{a_I}(|\chi\rangle\langle\chi|) + \frac{p^2}{N}I. \tag{14}$$

It follows that $s_{b_I} = s_{b_{II}} = \frac{2(1-p)}{N}(p = 1-q)$. In the range of $\frac{1-\sqrt{3}+\sqrt{2\sqrt{3}}}{2} \leq p \leq 1$, $s_{b_I} = s_{b_{II}} \leq \frac{1}{\sqrt{3}}$. So the maximum value of closeness which can be achieved by this process is $\frac{1}{\sqrt{3}}$ as in (Bandyopadhyay *et al.*, 1999; Ghosh *et al.*, 2000; Zhou and Guo, 2000). Of course, the maximum value $\frac{1}{3}$ of s can be achieved in the quantum machine by copying only one qubit.

According to the viewpoint in Cerf (1998, 2000), it is observed that the qubits a_I (a_{II}) and b_I (b_{II}) emerge from depolarizing channels of probability $P = 3\frac{(1-p)^2}{2N}$ and $P' = 3\frac{p^2}{2N}$, respectively. Hence the variation of the parameter p changes the capacities of two quantum channels such that the quantum correlation of the initial entangled state $|\chi\rangle$ is filtered in the branch $b_I b_{II}$ but partially kept in the branch $a_I a_{II}$. Therefore by copying both two qubits asymmetrically it is possible to disentangle an entangled state and keep some entanglement in the residual state. The fidelity of the output residual entangled state with respect to the input entangled state is state-dependent. While the scaling parameter s, which can be achieved by the proposed quantum machine, has the same range as in the work (Bandyopadhyay *et al.*, 1999; Ghosh *et al.*, 2000; Zhou and Guo, 2000).

On the other hand, but importantly, if we further analyze disentanglement and copying entanglement by analogy, the connection between copying (cloning and broadcasting) and disentanglement is noted (Mor, 1999). It is found that the conditions of perfect disentanglement into product states corresponds to cloning of one of the subsystems while the conditions of perfect disentanglement into separable states corresponds to broadcasting of one of the subsystems (Mor and Terno, 1999). And comparing the schemes of disentanglement and broadcasting entanglement by local symmetric cloning (Bandyopadhyay *et al.*, 1999; Bandyopadhyay and Kar, 1999), one can notice that for disentanglement the reduction factor η describing the quality of symmetric cloner satisfies $\eta \leq \frac{1}{\sqrt{3}}$, whereas for broadcasting it needs $\eta \geq \frac{1}{\sqrt{3}} (\eta = \frac{2}{3})$ for the optimal quantum cloner (Bruss *et al.*, 1998). In the meantime, disentanglement erases the quantum correlation (inseparability) between subsystems as broadcasting needs to retain the quantum correlation between subsystems, except that in both process the output-reduced-density matrix of each subsystem is as close as possible to the corresponding input-reduced-density matrix. There is an intuitive understanding that the disentangling state of an entangled state can be viewed as a separable copy of it. This suggests that it is possible to look for a unified way of dealing with broadcasting and disentangling an entangled state simultaneously. When associating with the actions of the above quantum machine which disentangles an entangled state and retains it nearly, it may be conjectured that copying both two qubit in an entangled state by an asymmetric $1 \rightarrow N(N > 2)$ cloning be such a single evolution that deals with broadcasting and disentangling an entangled state simultaneously.

To conclude, in the above mentioned, we have proposed a quantum machine, which for an input entangled state produces a disentangled state and almost retains the entangled state. The machine is based on an asymmetric $1 \rightarrow 2$ cloning. The flow of information in the cloning process is controlled by varying the parameter p so that the quantum entanglement is partially retained in one copy of the entangled state but erased in another. If using the $1 \rightarrow 2$ asymmetric telecloning in the quantum machine, we can distantly send a copying state of the entangled state to a receiver and a disentangling state of it to another according to the requirement of information distribution. Otherwise, if exploiting an asymmetric $1 \rightarrow N(N > 2)$ cloning to copy both qubits in an entangled state, a disentangling state and some copying states of the entangled state can be possible obtained simultaneously. The result shows that the disentanglement and broadcasting can be possibly combined in a single unitary evolution. We hope that it is helpful for understanding entanglement and useful for further studying quantum information and quantum computation.

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